

Stability Analysis for Reconfigurable Systems with Actuator Saturation

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Abstract

This paper discusses a combined analytic and simulation-based approach to assessing the stability of a control law in a system that may be subject to actuator saturation due to failures and subsequent reconfiguration. The analysis is based on linearized plant dynamics, a linearized state-feedback description of the nonlinear controller dynamics, and a nonlinear actuator model. For systems of this type, a method has recently been developed that provides less conservative estimates of the domain of attraction than other available methods. Domain of attraction estimates using this method are obtained by solving optimization problems with linear matrix inequality constraints, and are thus practical to obtain numerically. The domain of attraction estimates are used to guide simulation based stability analysis. The combined analytic and simulation based stability assessment approach is implemented in *RASCLE*¹, a software package designed to interface with an arbitrary C, C++, or FORTRAN simulation. Through the combination of analytic stability estimates and automated simulation-based analysis, *RASCLE* can efficiently provide information about the stability of the full nonlinear system under a wide range of conditions for the purpose of validating a reconfigurable controller.

1 Introduction

Reconfigurable flight control systems have the potential to greatly improve the safety of both military and civil aircraft by allowing the flight control system to con-

tinue to provide the pilot with good handling qualities even after failures have occurred. Certification of such systems is difficult, however, and presents a significant barrier to their implementation in fleet aircraft. Tools for validating the stability and robustness of reconfigurable control laws are needed for the laws to enter mainstream use and realize their potential safety benefits. *RASCLE* is one such software package designed to aid engineers in assessing the robustness of control laws [3].

The primary purpose of *RASCLE* is to automate intelligent simulation-based assessment of control law robustness. It is impossible to prove that even a linear controller will always be stable in a real application, or even in a high-fidelity simulation. The currently accepted practice for validating a control law is to combine closed form analysis with simulation based analysis, which is often not automated, to achieve confidence in the robustness of the control law. For advanced nonlinear, multivariate control laws, the testing requirements for certification will be significantly increased. The authors believe that a tool such as *RASCLE* that automates and add intelligence to Monte Carlo and systematic simulation-based searches will therefore be essential to certification of complex nonlinear and reconfigurable control laws.

One way to intelligently guide the simulation-based searches is to begin with an analytic estimate of the stable region for the control law. Though it is generally not possible to prove the stability of a control law in a high fidelity nonlinear simulation (hence the need for simulation-based stability assessment), it may be possible to prove stability for some simplified system. If the simplified system captures some key nonlinear behavior of the full simulation, such a stability proof may provide significant insight into the behavior of the full simulation.

¹Robustness Analysis for Simulation-based Control Law Evaluation

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One important nonlinearity encountered in aircraft is actuator saturation. This is of particular concern for reconfigurable systems following a failure, when using all of the remaining actuation capability may be necessary to maintain stability. Improved techniques have recently been developed for obtaining domain of attraction estimates for linear systems subject to actuator magnitude saturation [4]. The domain of attraction estimates generated by this method for systems with actuator magnitude saturation are leveraged by *RASCLE* to provide insight into the behavior of the full nonlinear system and to guide simulation-based searches.

In this paper, we discuss how the analytic domain of attraction estimates are generated, and how they are incorporated in the *RASCLE* software tool to help guide simulation-based searches. We then briefly illustrate the use of *RASCLE* to assess a reconfigurable control law that is being developed by Barron Associates, Inc. (BAI), and present results obtained by applying *RASCLE* to this control law. Future plans for *RASCLE* are discussed, and conclusions from this paper are presented.

2 Theoretical Stability Analysis

The domain of attraction estimation techniques currently employed in *RASCLE* apply to systems of the form

$$\begin{aligned} \dot{x} &= Ax + B\sigma(u) \\ u &= Fx, \end{aligned} \quad (1)$$

where $x \in \mathcal{R}^n$, $u \in \mathcal{R}^m$, and $\sigma(\cdot)$ represents the unity level saturation function. The key result of [4] is recalled in Theorem 2.1, which provides a condition under which a set is contractively invariant and thus inside the domain of attraction. The theorem makes use of the following definitions. Let $P \in \mathcal{R}^{n \times n}$ be a positive-definite matrix. Denote the ellipsoid

$$\Omega(P, \rho) = \{x \in \mathcal{R}^n : x^T P x \leq \rho\}.$$

Also, let $\mathcal{V} = \{\nu \in \mathcal{R}^m : \nu_i = \{0, 1\}\}$ and define $M(\nu, F, H) = \text{diag}(\nu)F + (I - \text{diag}(\nu))H$.

Theorem 2.1 *Given an ellipsoid $\Omega(P, \rho)$, if there exists an $H \in \mathcal{R}^{m \times n}$ such that*

$$(A + BM(\nu, F, H))^T P + P(A + BM(\nu, F, H)) < 0, \forall \nu \in \mathcal{V}$$

and $|h_i x| \leq 1, \forall x \in \Omega(P, \rho), i = [1, m]$, then $\Omega(P, \rho)$ is a contractively invariant set of the system (1).

Based on this theorem, an optimization problem with LMI constraints can be formulated to obtain a domain of attraction estimate that is less conservative than that

provided by other available techniques (refer to [4] for the proof of Theorem 2.1, and details of the LMI optimization problem).

Clearly, the domain of attraction estimation approach based on the preceding theorem is applicable only to systems with the specific structure given by (1). An aircraft model and the associated control system will, in general, be time varying and will contain nonlinearities in addition to the actuator saturation nonlinearity. To apply the domain of attraction analysis, we must first approximate the full aircraft model with a model in the form of (1). While a single such model will not suffice for the full flight envelope, it can often provide a reasonable approximation for a specific flight condition. We thus follow current practice and identify each trim condition of interest and linearize the plant and controller about this operating point. After adding appropriate actuator limits, we obtain a system in the form of (1) for each trim condition.

Having obtained system models at each trim point, we can apply the theoretical techniques to obtain an estimate of the stable region for the system about each trim point. The region obtained as a domain of attraction estimate is proven to be stable for the system of the form (1). Though stability of the complete nonlinear simulation model cannot be proven, if actuator saturation is one of the most important nonlinearities influencing the boundary of the domain of attraction, the theory should provide a reasonable estimate of the domain of attraction for the full system about each operating point.

3 Guiding Simulation-based Search

While the domain of attraction provides important information about the stability of a system, it is not a metric that is as widely referenced, or as familiar to many engineers as gain or phase margin. Because of their wide use, gain and phase margins are perhaps the most important outputs generated by *RASCLE*. A major challenge in developing *RASCLE* has been finding a way to link the important stability information provided by the domain of attraction with the widely accepted gain and phase margin metrics. This section describes the capabilities of *RASCLE* in terms of generating gain and phase margins, and how information from analytically generated domain of attraction estimates is leveraged to generate more meaningful simulation-based gain and phase margins.

RASCLE has the capability to search for single gain and phase margins, and for margins on pairs of simultaneous gains and delays. We refer to the single signal gain and phase margins as 1-dimensional stability bounds, and to the gains and delays introduced in these

signals as uncertainties. The margins for pairs of simultaneous gain and delay uncertainties are referred to as 2-dimensional stability boundaries. To compute a 1-dimensional bound, the user configures the simulation to run a specific maneuver. The tool runs the simulation repeatedly with this maneuver and with varying magnitudes of the uncertainty of interest. A boundary for the magnitude of the uncertainty is identified below which the system remains stable and above which it becomes unstable. A 2-dimensional boundary is computed similarly with two uncertainties being varied simultaneously. The specific search method used to efficiently find the boundary is beyond the scope of this paper.

An important consideration in computing the 1-D and 2-D stability boundaries is the nature of stability for linear and nonlinear systems. For a linear system, stability is a global property, and the 1-D and 2-D stability boundary results will not depend on the maneuver chosen to excite the system. High-fidelity aircraft simulations are nonlinear, however, and the level of a given uncertainty that the system can tolerate while remaining stable may depend on the direction and magnitude of the maneuver used to excite the system. Selection of a maneuver thus becomes an important part of the process of determining 1-D and 2-D stability boundaries. In addition to depending on the chosen maneuver, the stability of an aircraft will also depend on the chosen trim condition.

Because of the potentially important impact of the choice of trim condition and maneuver on the gain and phase margin results, *RASCLE* provides the capability to intelligently select from a user-provided set the trim condition and maneuver combination on which to focus its analysis. *RASCLE* attempts to identify the trim condition and maneuver combination for which the simulation will become unstable with the smallest level of added uncertainties, leading to the most conservative gain and phase margins. It is in selecting this worst case trim condition and maneuver combination that *RASCLE* leverages the domain of attraction results.

For each trim condition and maneuver combination supplied by the user, *RASCLE* runs the simulation under nominal conditions and records the state trajectory. Each nominal state trajectory is then scored based on the domain of attraction estimate for the corresponding trim condition. This score is computed as

$$\text{trajectory score} = \max_{t \in [0, T]} x(t)^T P x(t)$$

where $\Omega(P, 1) = \{x : x^T P x < 1\}$ is the domain of attraction estimate for the corresponding trim point, $x(t)$ is the recorded state vector at time t , and T is a user defined time. The trim condition and maneuver

combination corresponding to the trajectory with the highest score is selected by *RASCLE* as the one likely to tolerate the smallest levels of uncertainty before instability occurs, and is used in subsequent simulation-based stability analysis. The subsequent simulation-based analysis generally includes computation of 1-D and 2-D stability boundaries, and may include random or systematic searches of the uncertainty space, which are also provided as options in *RASCLE*. The user may, of course, elect to conduct additional simulation-based analyses of other trim condition and maneuver combinations.

4 F-18 Retrofit Reconfigurable Controller

The authors sought to demonstrate the capabilities of *RASCLE* by applying it to a current research problem that could benefit from the results of the analysis. BAI is actively working on several research efforts to develop fault-tolerant adaptive control systems, which offer a number of opportunities to apply the *RASCLE* software. The authors chose the development of a Retrofit Reconfigurable Control System (RRCS) for the F-18 as an effort to which the results of *RASCLE* are particularly relevant.

The goal of fault-tolerant adaptive control systems is to achieve desired closed-loop performance in the presence of unexpected dynamical behavior of the system. In the context of flight control, where these unpredicted responses might arise from airframe damage, sensor and actuator failures, or effector impairments, such technology offers tremendous benefits. For this reason, a great deal of research has been conducted by organizations including BAI to develop control approaches that can compensate for unforeseen anomalies. A number of these control techniques have performed well in high-fidelity numerical simulations and fixed-based pilot-in-the-loop experiments. A few of the approaches, including the Self Designing Controller developed by BAI, have even been successfully demonstrated in flight [5]. Yet, most of these designs are not practical for use in existing aircraft because they require a complete redesign of the hardware (sensors, flight computers, etc.) and software (control system).

The goal of the Retrofit Reconfigurable Control System being developed for the F-18 is to provide a retrofit capability by developing and implementing low-cost add-on modules for existing controllers that:

- provide reconfiguration to unmodeled dynamics, actuator failures, airframe damage, and control surface failures
- adapt or reconfigure the production controller only to the extent that the closed-loop system

is behaving unexpectedly

- minimize or eliminate the need to modify the production controller directly
- obviate requirements for additional hardware and sensors
- preserve the verification and validation (V&V) results that have been compiled for the baseline controller

Given the constraint that the structure of the baseline controller is preserved, one must affect reconfiguration by modifying either the inputs to or the outputs from the production control law. The Retrofit Reconfigurable Control System (RRCS) currently being developed by BAI takes the approach of modifying the inputs to the production control law.

Because the retrofit approach being developed preserves the baseline controller intact, the V&V process should benefit significantly from the results previously obtained for the baseline controller. Clearly, however, V&V of the retrofit controller is still a significant hurdle to implementation of the techniques in fleet aircraft. Because BAI has the goal of flight testing, and ultimately implementing the RRCS in fleet aircraft, this system provides a perfect example of the type of advanced control system whose development and certification can benefit from application of *RASCLE*.

5 Application of *RASCLE*

This section presents results from the application of *RASCLE* to analyze the RRCS described in the preceding section. The results presented here are not intended to represent a complete analysis of the stability of the RRCS and how it compares to the baseline controller, but rather to provide examples of the analysis capabilities and outputs of *RASCLE*.

To more fully tax the capabilities of the RRCS, a destabilizing change in the aerodynamic properties of the aircraft was introduced by reversing the effects of angle of attack on the aircraft pitching moment. The single trim condition selected for evaluation was 60 percent fuel, 20,000 ft, and Mach 0.7. A linear model of the aircraft dynamics was computed with the states $[P, Q, R, \alpha, \beta]$ (roll rate, pitch rate, yaw rate, angle of attack, and angle of sideslip). A linear state feedback approximation to the baseline controller was also computed, and based on these models and the method of [4], *RASCLE* computed the domain of attraction esti-

mate $\Omega(P, 1)$ with

$$P = \begin{bmatrix} 1.130 & -0.010 & 0.002 & -0.008 & 0.004 \\ -0.010 & 0.698 & 0.006 & 0.001 & 0.002 \\ 0.002 & 0.006 & 0.758 & -0.273 & 0.073 \\ -0.008 & 0.001 & 0.273 & 0.678 & -0.038 \\ 0.004 & 0.002 & 0.073 & -0.038 & 0.015 \end{bmatrix}.$$

This domain of attraction estimate was then used as described in Section 3 to select a maneuver for subsequent simulation-based analysis. Two maneuvers were considered. The first consisted of a square pulse in the pilot input on the pitch stick, and the second consisted of a square pulse in the pilot input on the roll stick. Based on the domain of attraction method for scoring trajectories described in Section 3, *RASCLE* selected the roll maneuver as the one expected to tolerate the smallest levels of instability, and thus the one to use for further analysis.

Table 1: 1-D Bounds for Baseline Controller (Pitch Maneuver)

Control Surface	Uncertainty	Stable Boundary	Unstable Boundary
Left Aileron	Gain		> 10
	Delay (s)		> 1
Left TEF	Gain	6.20	6.27
	Delay (s)		> 1
Left LEF	Gain		> 10
	Delay (s)		> 1
Left HT	Gain		> 10
	Delay (s)		> 1
Left Rudder	Gain		> 10
	Delay (s)		> 1

For comparison, the maneuver selection was also performed a second time using an alternate simulation-based ranking method that focuses on 1-D stability boundaries. The uncertainties considered in this paper are gains and delays on the actuator commands to the left horizontal tail (Left HT), left aileron, left leading edge flap (Left LEF), left trailing edge flap (Left TEF), and left rudder. *RASCLE* was configured to introduce gains on these signals in the range 1 to 10 and delays in the range 0 to 1 sec. Tables 1 and 2 show the 1-D stability boundaries for the closed loop system with the baseline controller for the pitch and roll maneuvers, respectively. As expected from inspection of these results, the 1-D stability boundary ranking method used by *RASCLE* selects the same (roll) maneuver chosen by the domain of attraction based approach. After using *RASCLE* to select the maneu-

Table 2: 1-D Bounds for Baseline Controller (Roll Maneuver)

Control Surface	Uncertainty	Stable Boundary	Unstable Boundary
Left Aileron	Gain	> 10	
	Delay (s)	> 1	
Left TEF	Gain	3.67	3.74
	Delay (s)	> 1	
Left LEF	Gain	> 10	
	Delay (s)	> 1	
Left HT	Gain	7.54	7.61
	Delay (s)	> 1	
Left Rudder	Gain	> 10	
	Delay (s)	> 1	

ver for further simulation-based analysis, the authors compared the performance of the baseline controller to that of the retrofit controller. Table 3 shows the 1-D stability boundaries for the system with the RRCS enabled and using the roll maneuver. This table reveals that the RRCS reduced the sensitivity of the baseline controller to gains on the trailing edge flap command, though sensitivity to gains on the left horizontal tail command did increase relative to the baseline. To ver-

Table 3: 1-D Bounds for Retrofit Controller (Roll Maneuver)

Control Surface	Uncertainty	Stable Boundary	Unstable Boundary
Left Aileron	Gain	> 10	
	Delay (s)	> 1	
Left TEF	Gain	> 10	
	Delay (s)	> 1	
Left LEF	Gain	> 10	
	Delay (s)	> 1	
Left HT	Gain	3.4609375	3.53125
	Delay (s)	> 1	
Left Rudder	Gain	> 10	
	Delay (s)	> 1	

ify the results reported by *RASCLE* and to better understand the effects of adding gain to the horizontal tail command, the authors used *RASCLE* to produce time series output data for the system with varying gain levels on the horizontal tail command. *RASCLE* first detected instability based in the pitch rate. Figs. 1

and 2 show plots of pitch rate response for the baseline controller and the RRCS for gains of 3.25 and 4.15, respectively. These gains fall slightly above and below the reported stability boundary for the RRCS. The

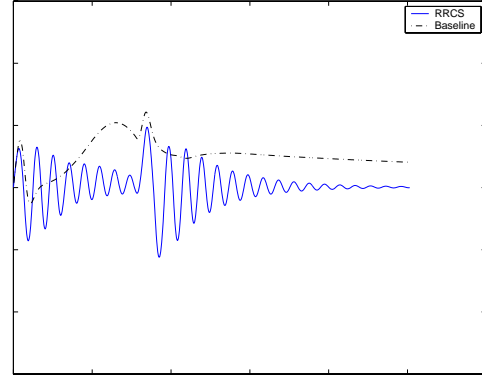


Figure 1: Q Response to Roll Maneuver, Left Horizontal Tail Command Gain = 3.25

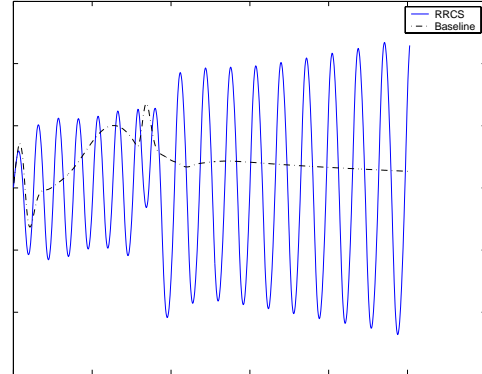


Figure 2: Q Response to Roll Maneuver, Left Horizontal Tail Command Gain = 4.15

plots verify that *RASCLE* has correctly identified the stability boundary for the reconfigurable controller and the destabilized F-18 dynamics. Had the gain margin been less than 6dB, we might have chosen to reduce the gains of the RRCS and settle for reduced tracking performance in exchange for increased stability; however, the *RASCLE* analysis indicates that the gain margin is still in excess of 10dB, and performance analysis conducted independent of *RASCLE* indicates that the RRCS provides greatly improved responses in the presence of actuator failures.

Figs. 3 and 4 show example 2-dimensional stability boundary curves generated by *RASCLE*. Fig. 3 shows the result that might be anticipated based on the 1-D stability boundary results. For the baseline controller, 1-D stability boundaries were found in the search range for both signals considered. The 2-D boundary curve indicates that when the two uncertainties are applied simultaneously, instability occurs somewhat before the

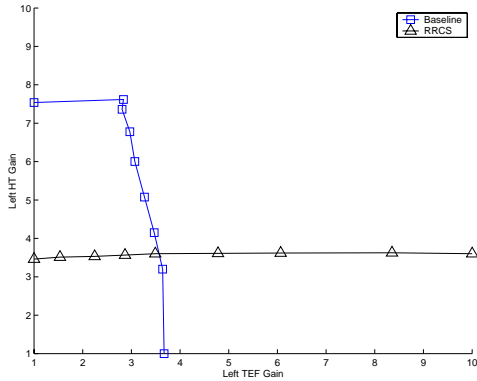


Figure 3: Left Horizontal Tail Command Gain vs. Left Trailing Edge Flap Command Gain Stability Boundary

individual uncertainty levels reach their 1-D bounds. In the case of the RRCS, no 1-D boundary was found for the left trailing edge flap command gain, and the 2-D boundary reveals that varying levels of this uncertainty have little effect on the level of the left horizontal tail command gain that leads to instability.

Fig. 4 shows a case in which the 2-D stability boundary provides insight into the system behavior that is not revealed by the 1-D boundaries. *RASCLE* indicates that

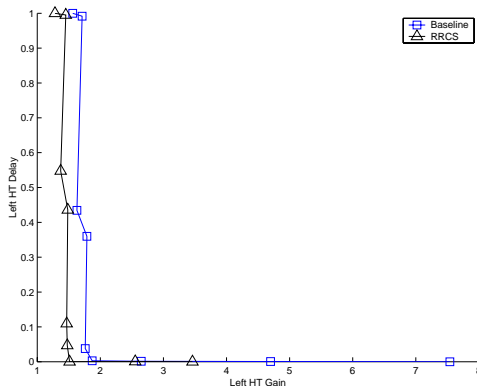


Figure 4: Left Horizontal Tail Command Delay vs. Gain Stability Boundary

delays of up to 1 second in the left horizontal tail command signal do not result in instability, but introducing even a small amount of delay greatly reduces the level of gain that can be tolerated in the Left HT command signal before instability occurs.

6 Future Extensions to *RASCLE*

In addition to actuator magnitude saturation, actuator rate saturation can be a significant destabilizing effect in aircraft. Eq. 2 gives a first order model of an actuator

with rate and position saturation

$$\dot{v} = K_1 \sigma(K_2(K_3 \sigma(u) - v)), \quad (2)$$

where v is a state representing actuator position, u is the actuator command, and σ is the unity level saturation function. An arbitrary saturation level can be effectively achieved by appropriately selecting gains K_1, K_2, K_3 , and as K_2 becomes large this model approaches an ideal rate limited actuator. A system using this actuator model, a linear plant model, and a state feedback linear controller can be represented as a linear system with two nested saturations. The authors have recently extended the methods of [4] to allow analysis of such systems, and are working to incorporate these methods with *RASCLE* [2, 1]. The results of stability analysis for systems with actuator rate and position limits will be incorporated into *RASCLE* in a fashion similar to that described in Section 3 for systems with position limits only.

7 Conclusions

The combination of analytic and simulation-based techniques in *RASCLE* yields a powerful software tool to help engineers assess the robustness of control laws. The tool offers potential benefits at all stage of the control law design and certification process. During development, the tool provides an efficient means of finding areas of instability that the designers need to improve. Ultimately, the tool performs extensive final testing, providing a high degree of confidence in the stability of the control law throughout the operating regime. The authors believe that the information provided by *RASCLE* will be a significant asset in the certification process.

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