Model-Based Sensor and Actuator Fault Detection and Isolation

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Abstract

This work concerns the development of an analytical redundancy-based approach for detecting and isolating sensor, actuator, and component (i.e., plant) faults in complex dynamical systems, such as aircraft and spacecraft. The method is based on the use of constrained Kalman filters, which are able to detect and isolate such faults by exploiting functional relationships that exist among various subsets of available actuator input and sensor output data. A statistical change detection technique based on a modification of the standard generalized likelihood ratio statistic is used to detect faults in real time. The feasibility and efficacy of the approach is demonstrated through simulation in the context of a nonlinear jet engine control system.

Fault Detection and Isolation

Fault detection and isolation (FDI) represents a major branch of modern automatic control theory of great practical importance. Any control system is dependent upon the quality of the data that it receives (i.e., sensors) and execution of the commands that it issues (i.e., via actuators). Quite obviously, reliance on bad data or an erroneous system model (which encompasses actuators, sensors, and the plant itself) can potentially result in dysfunctional responses on the part of the control system.

Sensor reliability is a pervasive concern in a broad range of aerospace applications. As perhaps the most common example, accelerometers and rate gyros conventionally used in aircraft flight control are subject to bias and drift errors. In a significant number of other realms, such as high-temperature turbine engines and magnetic bearings, sensors are often the most failure-prone components in complex engineering systems, many of which operate in adverse environments. Although sensors often represent a small cost component of a large system, sensor anomalies can be pernicious since they can bring an otherwise properly functioning system to a complete standstill.

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Modern solution approaches for detecting sensor and actuator failures are based on *analytic redundancy*, which looks for consistency relationships among sets of plant input (actuator) and output (sensor) signals. This requires a mathematical model of the system, which is customarily formulated based on linearized state-space observer theory. Use of such simplified modeling methods, however, becomes increasingly difficult, if not altogether impractical, as the complexity of aerospace systems, and the control methodologies that accompany them, increases. The ideas presented in this paper focus on empirical, data-driven techniques for deriving analytic redundancy models for real-world systems.

Analytic Redundancy

The approach that is introduced below constitutes a generic methodology for FDI in dynamical systems (more specifically, state-space plant models containing actuators and sensors). It makes use of statistical change detection techniques (specifically, the *generalized likelihood ratio*) to detect instrument faults in real time and *constrained Kalman filters* for generating residuals and isolating faults that can potentially occur in sensors, actuators, and plant components. This particular approach, like most other modern FDI methods, is based on the fundamental underlying principle of analytic redundancy, the essence of which is introduced by reference to Fig. 1.

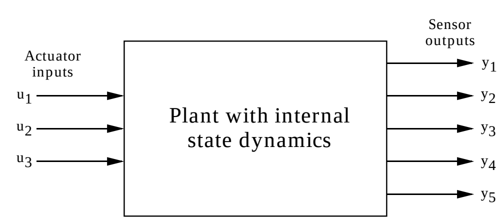

Figure 1: Generic Plant Model with Actuator Inputs and Sensor Outputs

Fig. 1 shows a generic multi-input multi-output (MIMO) plant model, whose inputs are actuator commands (u_1, u_2, $\ldots$) and whose outputs are sensor measurements (y_1, y_2, $\ldots$). Although these signals represent diverse, dissimilar variables, they are all interrelated through the common underlying state-space dynamics of the plant. From an information-theoretic signal processing perspective, this is significantly different from traditional *hardware redundancy* solutions, which rely on multiple sensors that nominally measure the same variable (e.g., angle of attack) and are usually based on majority voting logic. Analytic redundancy, by contrast, does not create any need for additional instrumentation hardware.

Implementation of analytic redundancy involves constructing *observers* for estimating the internal states of the plant and using the resulting state estimates to synthesize *predicted* sensor outputs (henceforth referred to as *surrogate signals*), which are different from *actual* sensor outputs. In general, the vector, $\hat{\underline{y}}_k$, of surrogate outputs at (discrete) time k is a function (explicitly or implicitly) of past actuator inputs ($\underline{u}_{k-1}$, $\underline{u}_{k-2}$, ...) and actual sensor outputs ($\underline{y}_{k-1}$, $\underline{y}_{k-2}$, ...). Surrogate signals therefore represent one-step-ahead predictions.[1] The computational form of this predictor function can be derived analytically by appealing to a mathematical model of the system (as is the case with Kalman filters) or empirically based on system identification techniques.

If the mathematical input-output model is accurate, then the surrogate signals should all be in close agreement with the corresponding actual signals, save for expected baseline levels of discrepancy due to noise (random plant disturbance or measurement noise) and/or modeling error. Faults in the actuators, sensors, or the plant itself become manifest through discrepancies that are abnormally large in a statistically significant sense. Statistical change detection (SCD) techniques are concerned with the quick detection of the onset of such departures from normalcy in a statistically rigorous manner.

Fault *isolation* is concerned with ascertaining which particular component (i.e., sensor, actuator, or plant) is most likely responsible for any discrepancies between the $\underline{y}$'s and $\hat{\underline{y}}$'s. The basic isolation strategy in the FDI approach that we advocate herein is to develop a bank of surrogate signal synthesizers (i.e., Kalman filters) that deliberately rely only on certain *subsets* of available actuator and sensor signals as inputs. For example, isolation of a fault in the first actuator (u_1) in Fig. 1 would require a dedicated Kalman filter that relies solely on the other instrument signals as inputs: (u_2, u_3, y_1, y_2, y_3, y_4, y_5). If a fault occurs that involves only the first actuator, the *relationships* among these other instrument signals (not necessarily the signal values themselves) remain normal. On the other hand, the relationships between u_1 and at least some other instrument signals will cease to be normal. Thus, any Kalman filter that relies on u_1 as a data source will yield anomalous (i.e., relatively large) residuals, whereas those that do not rely on u_1 will have normal (i.e., relatively small) residuals.

Similarly, isolation of a fault in, for example, the fifth sensor would require a dedicated Kalman filter that relies on the set (u_1, u_2, u_3, y_1, y_2, y_3, y_4) as inputs. Of course, two or more signals could be excluded if one wished to concentrate on certain compound fault scenarios (i.e., two or more simultaneous faults). Plant component faults will be revealed by anomalous residuals for all of the filters, since such faults represent a change in the underlying state-space model.

State-Space Modeling and Kalman Filtering

To explain our FDI methodology, we begin by focusing on generic systems whose dynamics can be represented by a discrete-time, linearized[2] state-space model:

$$\underline{x}_{k+1} = \boldsymbol{F}\underline{x}_k + \boldsymbol{G}\underline{u}_k + \underline{w}_{k+1} \qquad (1)$$

in which $\underline{x}_k$ denotes the vector of states at time k, $\underline{u}_k$ denotes the vector of deterministic plant inputs (e.g., actuation forces), and $\underline{w}_{k+1}$ is a stochastic vector of unmeasured plant disturbance noise. $\boldsymbol{F}$ and $\boldsymbol{G}$ are the discrete-time state transition and actuator matrices respectively. Sensor measurements are governed by the equation:

$$\underline{y}_k = \boldsymbol{C}\underline{x}_k + \underline{v}_k \qquad (2)$$

in which $\boldsymbol{C}$ is the sensor matrix and $\underline{v}_k$ is a stochastic vector of measurement noise.

Kalman filters use a simple predictor-corrector technique for estimating the plant states. The first step is a one-step-ahead prediction of the plant states, *viz.*,

$$\hat{\underline{x}}_{k+1}^{[k]} = \boldsymbol{F}\hat{\underline{x}}_k^{[k]} + \boldsymbol{G}\underline{u}_k \qquad (3)$$

in which $\hat{\underline{x}}_k^{[k]}$ denotes the estimate of $\underline{x}_k$ given all actuator and sensor data available through time k. $\hat{\underline{x}}_{k+1}^{[k]}$ denotes the estimate of $\underline{x}_{k+1}$ given data available through time k. The second step is a correction, which is used to adjust $\hat{\underline{x}}_{k+1}$ as soon as sensor measurements at the next time step become available, *viz.*,

$$\hat{\underline{x}}_{k+1}^{[k+1]} = \hat{\underline{x}}_{k+1}^{[k]} + \boldsymbol{g}(\underline{y}_{k+1} - \boldsymbol{C}\hat{\underline{x}}_{k+1}^{[k]}) \qquad (4)$$

in which $\boldsymbol{g}$ is an optimizable updating gain matrix. Note that the filter *innovation* (i.e., the factor to which the updating gain is applied) is the discrepancy between the actual sensor measurement, $\underline{y}_{k+1}$, and the surrogate signal, $\boldsymbol{C}\hat{\underline{x}}_{k+1}^{[k]}$.

It can be shown that the above equations result in the following simple dynamical model for the state estimation error, $\epsilon_k \equiv \underline{x}_k - \hat{\underline{x}}_k^{[k]}$, *viz.*,

$$\underline{\epsilon}_{k+1} = (\boldsymbol{1} - \boldsymbol{gC})\boldsymbol{F}\underline{\epsilon}_k + (\boldsymbol{1} - \boldsymbol{gC})\underline{w}_{k+1} - \boldsymbol{g}\underline{v}_{k+1} \qquad (5)$$

[1] Variants of the standard Kalman filter make it possible to predict more than one step ahead or to make retrospective state estimates.

[2] There are at least two techniques through which this formalism can be extended to nonlinear systems, one of which is demonstrated below.

Squaring both sides of Eq. 5 and taking the stochastic mean yields:

$$\begin{aligned}\boldsymbol{\Sigma}_{\epsilon\epsilon} &= (\mathbf{1}-\boldsymbol{g}\boldsymbol{C})\boldsymbol{F}\boldsymbol{\Sigma}_{\epsilon\epsilon}\boldsymbol{F}^T(\mathbf{1}-\boldsymbol{g}\boldsymbol{C})^T + \\ &\quad (\mathbf{1}-\boldsymbol{g}\boldsymbol{C})\boldsymbol{\Sigma}_{ww}(\mathbf{1}-\boldsymbol{g}\boldsymbol{C})^T + \boldsymbol{g}\boldsymbol{\Sigma}_{vv}\boldsymbol{g}^T \quad (6)\end{aligned}$$

in which $\boldsymbol{\Sigma}_{\epsilon\epsilon}$ is the covariance of the state estimation error, $\boldsymbol{\Sigma}_{ww}$ is the covariance of the plant disturbance noise, and $\boldsymbol{\Sigma}_{vv}$ is the covariance of the measurement noise. It is usually assumed that the plant disturbance and measurement noise are uncorrelated, but that assumption is not necessary mathematically. It is apparent that if the gain, $\boldsymbol{g}$, is given, the error covariance, $\boldsymbol{\Sigma}_{\epsilon\epsilon}$, can be obtained explicitly from Eq. 6 (which is a *discrete Lyapunov equation*).

Taking the first-order variation of Eq. 6 with respect to the updating gain, it can be shown that the error covariance is extremized (in a sense that can be made precise) if the equation:

$$\begin{aligned}(\boldsymbol{F}\boldsymbol{\Sigma}_{\epsilon\epsilon}\boldsymbol{F}^T + \boldsymbol{\Sigma}_{ww})\boldsymbol{C}^T &= \\ \boldsymbol{g}\left[\boldsymbol{C}(\boldsymbol{F}\boldsymbol{\Sigma}_{\epsilon\epsilon}\boldsymbol{F}^T + \boldsymbol{\Sigma}_{ww})\boldsymbol{C}^T + \boldsymbol{\Sigma}_{vv}\right] & \quad (7)\end{aligned}$$

is satisfied. Eqs. 6 and 7 furnish two equations in two unknowns, $\boldsymbol{g}$ and $\boldsymbol{\Sigma}_{\epsilon\epsilon}$. Joint iterative solution of the two equations makes it possible to solve for the optimal steady-state updating gain, $\boldsymbol{g}$, and the resulting error covariance, $\boldsymbol{\Sigma}_{\epsilon\epsilon}$.

Kalman Filters for FDI

The above formulation of the basic conventional Kalman filter assumes that all actuator and sensor signals are used as data sources for generating the state estimate and the resulting innovation sequences. Fault detection and isolation, however, require the use of special Kalman filters that make use of certain particular *subsets* of signals as inputs. There are multiple filters (one for each possible instrument that may potentially malfunction), which, in general, all have different updating gains. Derivation of these gains requires a reformulation of the theoretical analysis above.

As the first step in the rederivation, the actuation term in Eq. 1 must be partitioned into "included" and "excluded" inputs, *viz.*,

$$\underline{x}_{k+1} = \boldsymbol{F}\underline{x}_k + \boldsymbol{G}_{\text{incl}}(\underline{u}_{\text{incl}})_k + \boldsymbol{G}_{\text{excl}}(\underline{u}_{\text{excl}})_k + \underline{w}_{k+1} \quad (8)$$

Included inputs are used as data sources by the Kalman filter, whereas excluded inputs are not. In the case of the Kalman filter dedicated to isolating faults that may occur in the q'th actuator, for example, $\boldsymbol{G}_{\text{excl}}$ is simply the q'th column of the $\boldsymbol{G}$ matrix in Eq. 1. $\boldsymbol{G}_{\text{incl}}$ consists of all other columns of $\boldsymbol{G}$.

The predictor-corrector equations for the state estimation become:

$$\hat{\underline{x}}^{[k]}_{k+1} = \boldsymbol{F}\hat{\underline{x}}_k + \boldsymbol{G}_{\text{incl}}(\underline{u}_{\text{incl}})_k \quad (9a)$$

$$\hat{\underline{x}}_{k+1} = \hat{\underline{x}}^{[k]}_{k+1} + \boldsymbol{g}[\,(\underline{y}_{\text{incl}})_{k+1} - \boldsymbol{C}_{\text{incl}}\hat{\underline{x}}^{[k]}_{k+1}\,] \quad (9b)$$

in which the second equation emphasizes that only a subset of available sensor signals may be used for constructing the filter innovation. In the case of the Kalman filter dedicated to isolating faults that may occur in the r'th sensor, for example, $\boldsymbol{C}_{\text{incl}}$ is the $\boldsymbol{C}$ matrix in Eq. 2 with the r'th row deleted.

The error dynamics model (Eq. 5) becomes:

$$\begin{aligned}\underline{\epsilon}_{k+1} = (\mathbf{1}-\boldsymbol{g}\boldsymbol{C}_{\text{incl}})\boldsymbol{F}\underline{\epsilon}_k + (\mathbf{1}-\boldsymbol{g}\boldsymbol{C}_{\text{incl}})\boldsymbol{G}_{\text{excl}}(\underline{u}_{\text{excl}})_k + \\ (\mathbf{1}-\boldsymbol{g}\boldsymbol{C}_{\text{incl}})\underline{w}_{k+1} - \boldsymbol{g}\underline{v}_{k+1} \quad (10)\end{aligned}$$

from which it is evident that the excluded inputs appear on the right-hand side. The only way to make the error dynamics impervious to the excluded inputs is to require that the gain, $\boldsymbol{g}$, satisfy an equation of constraint, *viz.*,

$$(\mathbf{1}-\boldsymbol{g}\boldsymbol{C}_{\text{incl}})\boldsymbol{G}_{\text{excl}} = \mathbf{0} \quad (11)$$

The existence of a gain satisfying Eq. 11 requires that the matrices $\boldsymbol{C}_{\text{incl}}$ and $\boldsymbol{G}_{\text{excl}}$ jointly satisfy specific criteria. It can be shown that a necessary and sufficient mathematical condition is that the number of linearly independent rows $\boldsymbol{C}_{\text{incl}}$ be at least as great as the number of linearly independent columns of $\boldsymbol{G}_{\text{excl}}$. Heuristically, the implication is that it is possible to compensate for the non-measurement of the excluded input(s) if a sufficient number of sensor degrees of freedom, measuring appropriate state combinations, are utilized.

The above analysis implies that isolation of sensor faults is simpler than isolation of actuator faults. Whereas the former involves merely deleting rows of the $\boldsymbol{C}$ matrix, the latter represents a more complicated problem in that the constraint equation (Eq. 11) becomes nontrivial.

Constrained Kalman Filtering

Assuming that $\boldsymbol{C}_{\text{incl}}$ and $\boldsymbol{G}_{\text{excl}}$ jointly satisfy the aforementioned criterion, the general solution of Eq. 11 can be expressed as a linear subspace partitioning of the updating gain, *viz.*,

$$\boldsymbol{g} = \boldsymbol{g}_{\perp} + \boldsymbol{\gamma}\boldsymbol{g}_{\parallel} \quad (12)$$

The first term on the right-hand side is computed as:

$$\boldsymbol{g}_{\perp} \equiv \boldsymbol{G}_{\text{excl}}(\boldsymbol{C}_{\text{incl}}\boldsymbol{G}_{\text{excl}})^{+} \quad (13)$$

in which the '+' superscript denotes the matrix pseudo-inverse. As a theorem of matrix algebra, it can be shown that of all the gain matrices satisfying Eq. 11, $\boldsymbol{g}_{\perp}$ is the one with the smallest Euclidean norm.

$\boldsymbol{g}_{\parallel}$ is the left kernel space of $\boldsymbol{C}_{\text{incl}}\boldsymbol{G}_{\text{excl}}$ and, like $\boldsymbol{g}_{\perp}$, can be calculated by straightforward means. The $\boldsymbol{\gamma}$ matrix in Eq. 12 represents the freedom left in the gain after having satisfied the constraint condition. For cases in which no inputs are excluded (i.e., $\boldsymbol{G}_{\text{excl}}$ has zero columns), it can be shown that $\boldsymbol{g}_\perp$ is a zero matrix and $\boldsymbol{g}_\parallel$ is an identity matrix. $\boldsymbol{\gamma}$ then coincides with the ordinary Kalman gain in Eqs. 4-7.

The objective of the constrained Kalman filter derivation is to optimize $\boldsymbol{\gamma}$, such that the resulting error covariance is minimized. It can be shown that Eq. 6 remains nearly unchanged in the constrained Kalman filter formulation, *viz.*,

$$\boldsymbol{\Sigma}_{\epsilon\epsilon} = (\mathbf{1} - \boldsymbol{g}\boldsymbol{C}_{\text{incl}})\boldsymbol{F}\boldsymbol{\Sigma}_{\epsilon\epsilon}\boldsymbol{F}^T(\mathbf{1} - \boldsymbol{g}\boldsymbol{C}_{\text{incl}})^T + (\mathbf{1} - \boldsymbol{g}\boldsymbol{C}_{\text{incl}})\boldsymbol{\Sigma}_{ww}(\mathbf{1} - \boldsymbol{g}\boldsymbol{C}_{\text{incl}})^T + \boldsymbol{g}\boldsymbol{\Sigma}_{vv}\boldsymbol{g}^T \qquad (14)$$

As in the ordinary Kalman filter case, the next step is to take the first-order variation of Eq. 14, but now, only constrained variations, with respect to $\boldsymbol{\gamma}$, are admitted. The final result that emerges is:

$$\left[(\mathbf{1} - \boldsymbol{g}_\perp\boldsymbol{C}_{\text{incl}})(\boldsymbol{F}\boldsymbol{\Sigma}_{\epsilon\epsilon}\boldsymbol{F}^T + \boldsymbol{\Sigma}_{ww})\boldsymbol{C}_{\text{incl}}^T - \boldsymbol{g}_\perp\boldsymbol{\Sigma}_{vv} \right] \boldsymbol{g}_\parallel^T = \boldsymbol{\gamma}\boldsymbol{g}_\parallel \left[\boldsymbol{C}_{\text{incl}}(\boldsymbol{F}\boldsymbol{\Sigma}_{\epsilon\epsilon}\boldsymbol{F}^T + \boldsymbol{\Sigma}_{ww})\boldsymbol{C}_{\text{incl}}^T + \boldsymbol{\Sigma}_{vv} \right] \boldsymbol{g}_\parallel^T \qquad (15)$$

Joint iterative solution of Eqs. 14 and 15, with use of Eq. 12, enables one to solve for $\boldsymbol{\gamma}$ and $\boldsymbol{\Sigma}_{\epsilon\epsilon}$ simultaneously. Substituting the resulting $\boldsymbol{\gamma}$ into Eq. 12 yields the optimal constrained updating gain. For cases in which $\boldsymbol{G}_{\text{excl}}$ has zero columns, it can be seen that Eq. 15 reduces to Eq. 7, with $\boldsymbol{C}_{\text{incl}}$ in place of $\boldsymbol{C}$.

Implications of the Constrained Kalman Filtering Technique

The constrained Kalman filtering approach furnishes a very general methodology for detecting and isolating faults in dynamical systems, based on mathematical modeling of relationships between arbitrarily specified subsets of available input and output signals. It provides a unified theoretical framework for treating sensor, actuator, and plant faults. It is also readily applicable to scenarios involving compound faults, as well as single-instrument faults.

Fault isolation requires a bank of constrained Kalman filters, one for each instrument that is subject to possible malfunction. In the application example presented below, for example, there are six sensors and one actuator. For single-instrument faults, this implies a total of seven filters, the gains of which are all different and can be computed analytically. Such a filter bank will be able to detect and isolate any fault that is confined to a single instrument.

It should be noted that "input" signals are not necessarily limited to actuator commands. In modeling nonlinear plant dynamics, it is often useful to regard quadratic and higher-order terms in the state variables as "nuisance" perturbations which act, in effect, as virtual actuators. If the mathematical forms of these nonlinear terms are known, they can be treated as excluded inputs under the above formalism. The resulting FDI filters will then be impervious to nonlinearities in the plant dynamics model. Since linearized state-space models, especially in aerospace applications, typically represent simplified approximations of complicated models, this is of substantial interest from a robustness viewpoint.

Extended Constrained Kalman Filtering

The constrained Kalman filtering technique can readily be extended to encompass nonlinear plant dynamics models, *viz.*,

$$\underline{x}_{k+1} = \underline{x}_k + h\underline{f}\,[\,\underline{x}_k, (\underline{u}_{\text{incl}})_k, (\underline{u}_{\text{excl}})_k\,] + \underline{w}_{k+1} \qquad (16)$$

in which h is the time step size (which must be sufficiently small), $\underline{f}$ is the continuous-time state transition function (which must be differentiable), and $\underline{w}_{k+1}$ is a disturbance term.

The general *extended constrained Kalman filter* (ECKF) entails a two-step predictor-corrector technique for updating the plant state estimates, *viz.*,

$$\hat{\underline{x}}_{k+1}^{[k]} = \hat{\underline{x}}_k^{[k]} + h\underline{f}\left[\hat{\underline{x}}_k^{[k]}, (\underline{u}_{\text{incl}})_k, \underline{0}\right] \qquad (17a)$$

$$\hat{\underline{x}}_{k+1}^{[k+1]} = \hat{\underline{x}}_{k+1}^{[k]} + \boldsymbol{g}_{k+1}\left[(\underline{y}_{\text{incl}})_{k+1} - \underline{s}_{\text{incl}}(\hat{\underline{x}}_{k+1}^{[k]})\right] \qquad (17b)$$

In Eq. 17a, the uncorrected prediction, $\hat{\underline{x}}_{k+1}^{[k]}$, is calculated as if the excluded inputs are zero. In Eq. 17b, the correction makes use of a time-varying updating gain, $\boldsymbol{g}_{k+1}$. The notation $\underline{s}_{\text{incl}}(\cdot)$ signifies that the sensor output, in the most general scenario, is an arbitrary function of the states.

After some methodical algebra, it can be shown that the gain, $\boldsymbol{g}_{k+1}$, in Eq. 17b can be optimized via formulas that are very similar to those introduced in the preceding sections.

Jet Engine Control System Example

We now demonstrate application of the constrained Kalman filtering technique on a specific simulation problem that involves a jet engine control system [1]. The simulated plant model consists of three states:

$x_1 \equiv$ low-pressure rotor speed
$x_2 \equiv$ high-pressure rotor speed
$x_3 \equiv$ main fuel burner flow

one actuator:

$u_1 \equiv$ throttle (fuel flow) command

and six sensors (direct measurement of the states plus three others):

$y_1 \equiv x_1$	$y_4 \equiv$ compressor discharge pressure
$y_2 \equiv x_2$	$y_5 \equiv$ turbine discharge pressure
$y_3 \equiv x_3$	$y_6 \equiv$ turbine exit pressure

The plant dynamics are governed by the discrete-time state-transition equation:

$$\underline{x}_{k+1} = \boldsymbol{F}\underline{x}_k + \boldsymbol{G}\underline{u}_k + \boldsymbol{H}\underline{x}_k^2 + \underline{w}_{k+1} \tag{17c}$$

in which $\underline{x}_k^2$ denotes the 6×1 vector of quadratic state combinations, *viz.*,

$$\underline{x}^2 \equiv (\, x_1^2 \quad x_2^2 \quad x_3^2 \quad x_1x_2 \quad x_1x_3 \quad x_2x_3 \,)^T \tag{17d}$$

The dimensions of $\boldsymbol{F}$, $\boldsymbol{G}$, and $\boldsymbol{H}$ are respectively 3×3, 3×1, and 3×6. The sensor output is governed by Eq. 2, in which the sensor matrix, $\boldsymbol{C}$, is of dimension 6×3.

The FDI observer bank consists of six filters (herein labeled S1, ..., S6) dedicated to isolating sensor faults and one filter (herein labeled A1) dedicated to isolating actuator faults. In the former, the $\boldsymbol{C}_{\text{incl}}$'s are each of dimension 5×3. We also have $\boldsymbol{G}_{\text{incl}} = \boldsymbol{G}$ and $\boldsymbol{G}_{\text{excl}} = \boldsymbol{H}$. In A1, $\boldsymbol{C}_{\text{incl}}$ is the full 6×3 $\boldsymbol{C}$ matrix. $\boldsymbol{G}_{\text{incl}}$ is 3×0, and $\boldsymbol{G}_{\text{excl}} = [\,\boldsymbol{H} \quad \boldsymbol{G}\,]$, which is 3×7.

Simulation Results

The aforementioned jet engine system was simulated under several fault scenarios, representative results for two of which are shown in Fig. 2. The seven rows correspond to the FDI observers, S1-S6 and A1. There are seven ECKFs, one dedicated to each sensor or actuator component that can potentially fail.

Results plotted in Fig. 2 are those of a modified *general likelihood ratio* (GLR) statistic, S_k, as a function of time. This GLR statistic was developed by the authors to overcome the fact that the standard GLR statistic does not readily lend itself to a recursive implementation. The approach involved modifying the basic GLR statistic in such a way that only a finite number of trailing samples are retained in memory. The first column in Fig. 2 shows results for a scenario (Case 1) involving an additive sensor fault in S4. The second column shows results for a separate simulation run (Case 2) involving a multiplicative throttle actuator fault in A1.

In both simulation runs, an exponential throttle pulse is applied to the system at 0.40 sec., two-fifths of the way into the run. The faults occur at 0.52 sec., which is before the throttle pulse dies out. In Case 1, it is evident that the S4 residuals, which do not draw upon the bad sensor channel (y_4), are insensitive to the effects of the fault in sensor 4. Hence, the S4 detection statistic remains normal. All other detectors, however, detect the fault condition quickly, since their residuals draw upon y_4 and therefore become contaminated. In Case 2, it is evident that only the A1 detection statistic remains normal. All others detect the fault shortly after it occurs.

The results in Fig. 2 clearly demonstrate that successful fault detection and isolation are achieved using the

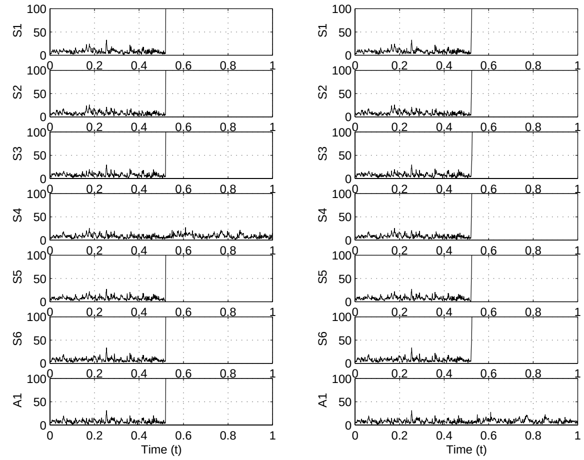

Figure 2: Modified GLR Results: Case 1 (left column) – Sensor Fault; Case 2 (right column) – Actuator Fault

ECKF method. As should be the case, all but one GLR residual goes bad quickly after the onset of a fault condition. To obtain these results, the use of time-varying $\widehat{\boldsymbol{F}}_k$ and time-varying updating gains were necessary.

Conclusions

The methodology presented in this paper concerns an algorithmic approach for detecting and isolating faults in complex dynamical systems, such as aircraft flight control systems and turbine engines. The method is based on the use of extended constrained Kalman filters, which are able to detect and isolate such faults by exploiting analytic redundancy that exists among various subsets of available actuator input and sensor output data. A statistical change detection technique based on a modification of the standard generalized likelihood ratio statistic is used to detect faults in real time. Simulation results for a nonlinear jet engine control system are used to demonstrate the feasibility and efficacy of the approach. Future work will focus on applying the approach in aircraft simulations and flight tests.

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